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Effect of Secondary Sterility on Open Birth Interval

1. Introduction

OPEN birth interval was proposed by Srinivasan (1967, 1968) as a summary measure of current fertility level of a married woman. However, the concept of closed and open birth intervals is well known in the renewal theory in the form of recurrence times or residual life-time of a unit at a specified time, Cox (1962). Particularly the open birth interval is quite close to backward recurrence time. Recently various objections have been raised against it and it has been observed that this is insensitive to the changes in fecundability at least for higher parties, (Venkatacharya, 1969 and Pathak, 1971).

The discussion suggests the need for a still deeper study of the mechanism underlying the variation in open birth intervals associated with different parities. The motivations behind its use as a measure of fertility are probably the facts that (1) the information needed in its calculation may be collected very easily without incurring non-sampling errors, and (2) a better alternative to it is not yet available in order to predict the current fertility level of a woman. For a fecund woman, open birth interval fails to reveal the exact fertility level if the point of the last live birth is very near to the observation period because its sensitivity is confounded with the truncation effect. It is, however, expected

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that in case of intervening secondary sterility, open birth interval may change significantly and may therefore be treated sensitive to the incidence of secondary sterility.

The aim of the present paper is to examine the effect of secondary sterility on open birth interval by means of a probability model derived, in Section 3 below, on the basis of some simple assumptions. This would indirectly suggest that the open birth interval should be used to examine the extent of recurrence of secondary sterility after each parity at time T from the marriage of a woman. Since age and parity of women are positively associated, it is convenient to associate the incidence of secondary sterility with parity rather than age as an approximation. The findings of the study have been summarised in Section 4. The values of the involved parameters of the proposed model have been taken arbitrarily but they are consistent with the empirical experience in India. We tiave, however, considered only non-contracepting women for assessing the impact of varying patterns of secondary sterility on the open birth interval.

3. Notations and Assumptions

(1) We denote by $(0, T)$ the total period of marital life enjoyed by a woman at the time of survey since the date of consummation of her marriage. The origin of this represents the effective date of marriage where both husband and wife start living together ;

(2) It is assumed that none of the women at time $T=0$ were pregnant;

(3) It is further assumed for the sake of simplicity that every conception leads to a birth, and the chance of miscarriage is zero, giving rise to a one-to-one correspondence between a pregnancy and a birth during the period $(0, T)$;

(4) Denoting by h the total infecundable exposure following a pregnancy, we assume h = a gestation period of 9 months + a period of post partum amenorrhoea of 6 months = 1.25 years and this is taken as constant, irrespective of the age of the woman and the order of the pregnancy. We also assume for simplicity that the length of this infecundable exposure does not vary from woman to woman;

(5) The number of coitions for a married woman during any arbitrary time-interval (t_1, t_2), $0 \leqslant t_1 \leqslant t_2 \leqslant T$, is a random variable and follows a Poisson distribution with the parameter $m(t_2 - t_1)$, where $m > 0$ is a constant. Thus the probability of having r coitions during interval $(t_2 - t_1)$ is given by

$$\frac{\{m(t_2 - t_1)\}^r}{r!} \exp[-m(t_2 - t_1)]\,; \tag{2.1}$$

(6) The coitions are mutually independent and p, the conditional probability for a coition at time t to result in a conception, is constant and is positive if there were no conception in the period $(t - h, t)$, $t \leqslant T$. Otherwise it is zero;

(7) The probability that a woman with parity $i \geqslant 0$ is fecund is α_i and that she is not so is $1 - \alpha_i$. Though it is very difficult to know when the incidence of secondary sterility occurs, we may assume that it occurs only after the end of the infecundable exposure associated with a pregnancy. This assumption may be treated as a first approximation to the real situation.

The above assumptions lead to the derivation of various forms of modified Poisson distribution which may be applied for different purposes, (*e.g.* Singh and Bhattacharya 1970; Pathak, 1971). The assumptions stated above are over-simplified but are supposed to give a close description of the reality in the first instance for studying the effect of secondary sterility on the open birth interval.

3. Development of the Model

Now let the woman be observed at time T from her marriage. Then the continuous *p. d. f.* of U_i (open birth interval after parity i at time T from the marriage date) for a women at time T is given by

$$g(U_i) = \left(\prod_{s=0}^{i} \alpha_s\right) f_1(U_i) + \left(\prod_{s=0}^{i-1} \alpha_s\right)(1 - \alpha_i) f_2(U_i)$$

$$\text{for } h \leqslant U_i \leqslant T - (i-1)h$$

$$= \left(\prod_{s=0}^{i-1} \alpha_s\right) f_2(U_i) \quad \text{for } 0 \leqslant U_i \leqslant h \tag{3.1}$$

where $$f_1(U_i) = k^i \frac{\{T - U_i - g - (i-1)h\}^{i-1}}{(i-1)!} \exp[-k(T - g - ih)]$$

$$f_2(U_i) = k^i \frac{\{T - U_i - g - (i-1)h\}^{i-1}}{(i-1)!} \exp[-k(T - U_i - g - (i-1)h)]$$

g = the length of the gestation period.

Proof

Let us assume that the married woman under study has i live-births during the period $(0, T)$ and the i-th child was born at time $T - U_i$ from her marriage so that U_i is the open birth interval at time T, the time of the observation. Let i successive conceptions occur at times $x_1, x_2, \ldots, x_i$ respectively. Obviously $x_i = T - U_i - g$. Since h is constant for all conceptions, x_{i-1} lies anywhere in the interval $\{(i-2)h, T - u_i - g - h\}$; x_{i-2} lies anywhere in the interval $\{(i-3)h, x_{i-1} - h\}; \ldots$; x_1 lies anywhere in the interval $(0, x_2 - h)$. Also since i conceptions occur successively at times $x_1, x_2, \ldots x_{i-1}, x_i$, no conceptions occur during the exposed periods $(0, x_1)$, $(x_1 + h, x_2)$, ... and $(x_{i-1} + h, x_i)$. Thus under our assumptions the probability that the first conception takes place during the period $(x_1, x_1 + dx_1)$ is given by the product of the probability that there is no conception during period $(0, x_1)$ and the probability that a conception takes place during the period $(x_1, x_1 + dx_1)$ i.e.

$$\exp[-kx_1].\, kdx_1 \tag{3.2}$$

where $$k = mp$$

Probability that there is no conception during the period $(x_1 + h, x_2)$ and the second conception occurs during $(x_2, x_2 + dx_2)$ is given by

$$\exp[-k(x_2 - \overline{x_1 + h})].\, kdx_2 \tag{3.3}$$

Similarly, the probability that there is no conception during the period $(x_j + h, x_{j+1})$, $j=2, 3, \ldots i-1$ and $(j+1)$-th conception occurs in the infinitesimal period $(x_{j+1}, x_{j+1} + dx_{j+1})$ is given by

$$\exp[-k(x_{j+1} - \overline{x_j + h})]\, kdx_{j+1}. \tag{3.4}$$

Hence the joint probability that i successive conceptions occur to the woman at times $x_1, x_2, \ldots, x_i$ from the date of her marriage is given by

$$\int_{x_{i-1}} \cdots \int_{x_2} \int_{x_1} \exp\left[- A\,(0, x_1) - A(x_1 + h, x_2) - \ldots - A(x_{i-1} + h, x_i)\right] k^i\, dx_1\, dx_2 \ldots dx_i \qquad (3.5)$$

where $\quad A(x_1, x_2) = \int_{x_1}^{x_2} k\, dx \qquad 0 \leqslant x_1 \leqslant x_2$

since $\quad x_i = T - U_i - g, \qquad dx_i = - du_i$

where du_i is negative. By integrating the above integral $x_1, x_2, \ldots x_{i-1}$, the probability that i successive births occur, the i-th birth occuring in the interval $(T - U_i, T - U_i + dU_i)$, is given by

$$[k^i \{T - u_i - g - (i-1)\, h\}^{i-1} / (i-1)\,!] \quad \exp\left[- k\, \{T - u_i - g - (i-1)\, h\}\right] du_i \quad (3.6)$$

(where du_i has been measured in the positive direction).

After the i-th conception has occurred at time $x_i = T - U_i - g$, under our assumption no conception can occur for a further period of length h and no birth other than the one following the pregnancy corresponding to the i-th conception can therefore occur till the end of the period $h + g$ from the point x_i. In such a case, U_i, the open birth interval at time T should not exceed h. Thus the probability that $U_i = u_i$, for $0 \leqslant u_i \leqslant h$ is equivalent to the expression given by (3.6), which may be expressed as $f_2(u_i)du_i$. If $U_i \geqslant h$, the probability that $U_i = u_i$ is given by the probability that i-th birth occurred at time $T - U_i$ and there were no conceptions in the interval of length $U_i - g$ of which only $U_i - h$ happened to be the length of the exposed period. The probability that woman does not conceive during the period of length $U_i - h$ is

$$\exp\ \left[- k\, (u_i - h)\right] \qquad (3.7)$$

Now multiplying (3.6) by (3.7), we get the probability that $U_i = u_i$, $(u_i \geqslant h)$ as

$$k^i \exp\left[- k(T - g - ih)\right] \ \left[\{T - u_i - g - (i-1)h\}^{i-1}/(i-1)\,!\right] du_i = f_1\,(u_i)\, dui \qquad (3.8)$$

The woman with parity i is either fecund or sterile. The probability that she is fecund is given by $\prod_{j=0}^{i} \alpha_j$ and that she is not so $\prod_{j=0}^{i-1} \alpha_j (1 - \alpha_i)$. Again in case a woman becomes secondary sterile after attaining parity i, there is no question of occurring another birth to her with probability one and therefore the probability of the event $U_i = u_i$ is given by $f_2(u_i) du_i$ for all U_i. Combining the arguments above we can therefore obtain easily the result (3.1) with the help of expressions (3.6) and (3.8).

Now let X denote the parity of the woman at time T from her marriage. Then it can be shown that

$$P[x = 0] = 1 - \alpha_0 + \alpha_0 \exp[-k(T - g)]$$

$$P[x = i] = \left(\prod_{s=0}^{i-1} \alpha_s\right) [1 - \alpha_i + \alpha_i \exp[-k(T - ih - g)] \left(\sum_{s=0}^{i} k^s (T - ih - g)^s / s!\right)]$$

$$- \exp[-k\{T - (i-1)h - g\}] \sum_{s=0}^{i-1} k^s (T - (i-1)h - g)^s / s!]$$

$$\text{for } i = 1, 2, \ldots, n - 1 \qquad (3.9)$$

$$P[x = n] = 1 - P[x \leqslant n - 1]$$

where n is the last parity which can be attained during the period $(0, T)$. The proof of the above result is very simple. The parameter 'k' may vary among couples but for simplicity we assume here that it does not vary from couple to couple.

4. Results

We have presented the mean open birth intervals associated with different parities along with the parity progression probabilities for $T = 5$, 10 and 15 years and $k = .40$ and .60. We have considered the values of $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ associated with parities 0, 1, 2, 3, . . . , n respectively as (.95, .95, .95, .95, ...95) ; (.95, .95, .95, .80, . . , .80) ; (.95, .95, .95, .25, .25, . . . , .25) forming levels 1, 2 and 3 respectively. The values of k seem to vary between .40 and .65 as suggested by some studies of Indian data, by Singh and Pathak (1968) and Singh and Bhattacharya (1970). We select two values .40 and .60 for k. However, these values for k may be treated as only arbitrarily selected for the study of the mechanism of open birth interval. Selection of values for $\alpha_0, \alpha_1, \ldots, \alpha_n$ is also arbitrary considering that after parity 3 the proportion of secondary sterile women would increase. Mean open birth intervals for different values of k and T and different levels of secondary sterility have been shown in Table 1. In Table 2 the parity progression probabilities have been shown.

Due to the incidence of secondary sterility we expect an increase in the mean open birth intervals associated with different parities assuming other parameters of the model to be constant.

Following are some of the salient points which emerge from this analysis :

1. The increase in the level of 'k'* causes increase in the mean open birth interval irrespective of the level of secondary sterility, parity and marriage duration. Also the increase in the level of secondary sterility causes a uniform increase in the value of mean open birth interval for all parities.

2. Parity has negative association with the open birth interval because the latter complements the sum of the preceding closed birth intervals for a fixed marital duration. In order to understand the mechanism underlying the variation in the open birth interval it would be useful to know the probabilities of attaining different parities by a woman under study. The probabilites of attaining different parities have been given in Table 2.

3. As expected, for all the parities and for all the levels of secondary sterility, the mean open birth interval increases as the duration of marriage increases. However, for higher parities the mean open birth intervals fail to show differences distinctively even if the marriage duration increases from 10 years to 15 years.

4. For extreme latter parities the mean open birth interval becomes insensitive either to the changes in fecundability or the pattern of secondary sterility. This may be due to two factors (*i*) very few women attain such higher parities, and (*ii*) with given pattern of fecundability

The paramenter k is related to the monthly chance of conception 'p' as p=l—exp [—*k/12]. This* p is different fram p given in the assumptions of Section 2 disussed earlier.

TABLE 1-MEAN OPEN BIRTH INTERVAL CORRESPONDING TO PARITY *i* FOR DIFFERENT VALUES OF *T* AND *k* AND DIFFERENT LEVELS OF SECONDARY STERILITY

T (Years)	k	Level of 'α'	Last Parity									
			1	2	3	4	5	6	7	8	9	10+
5	.40	1	2.2761	1.1136	.4846	.3778						
		2	. 2.2761	1.1136	.4848	*						
		3	2.2761	1.1136	.4857	*						
	.60	1	2.3595	1.1687	.5098	.4215						
		2	2.3595	1.1687	.5102	*						
		3	2.3595	1.1687	.5117	*						
10	.40	1	5.3293	2.4893	1.8875	1.2448	.8086	.4824	.2345			
		2	5.3293	2.9893	2.0262	1.2888	.8193	.4835	.2345			
		3	5.3293	2.989?	2.3954	1 4308	.8571	.4875	.2344			
	.60	1	6.3552	2.3362	2.0166	1.3207	.8610	.5119	.2429			
		2	6.3352	3.3362	2.2933	1.4033	.8802	.5138	.2429			
		3	6.3352	3.3362	2.8591	1.6435	.9462	.5209	.2429			
15	.40	1	10.1143	5.6655	3.5004	2.4092	1.7537	1.3044	.9683	.7013	.4812	.2987
		2	10.1145	5.6655	4.2880	2.7611	1.9063	1.3691	.9937	.7095	.4828	.2988
		3	10.1143	5.6655	5.4212	3.5149	2.3217	1.5725	1.0805	.7386	.4888	.2900
	.60	1	12.2644	8.0496	4.4942	2.7515	1.9046	1.3958	1.0343	.7497	.5123	.3135
		2	12.2644	8.0496	5.9104	3.5327	2.5269	1.0829	1.0829	.7648	.5153	.3137
		3	12.2644	8.0496	6.8863	5.5635	2.9244	1.S859	1.2386	.8175	.5261	.3142

*These values are based on rare events and are not very significant.

TABLE 2—PROBABILITY OF ATTAINING PARITY / FOR DIFFERENT VALUES OF *T*, *k* AND DIFFERENT PATTERNS OF SECONDARY STERILITY

T (Years)	K	Level of α_i	Parity 0	1	2	3	4	5	6	7	8	9	10+
5	.40	1	.22355	.47197	.27521	.02923	.00005						
		2	.22355	.47197	.27521	.02924	.00003						
		3	.22355	.47197	.27521	.02926	.00001						
	.60	1	.12418	.39103	.40786	.07671	.00022						
		2	.12418	.39103	.40786	.07675	.00018						
		3	.12418	.39103	.40786	.07687	.00006						
10	.40	1	.07349	.17852	.31384	.28702	.12422	.02181	.00110	.00001			
		2	.07349	.17852	.31384	.31025	.10766	.01559	.00066	*			
		3	.07349	.17852	.31384	.39543	.03714	.06157	.00001	*			
	.60	1	.05369	.08689	.19999	.31762	.25237	.08182	.00755	.00008			
		2	.05369	.08689	.19999	.37159	.22441	.05888	.00452	.00004			
		3	.05369	.08689	.19999	.56948	.08376	.00608	.00013	*			
15	.40	1	.05318	.07519	.14484	.23425	.25042	.16454	.06312	.01305	.00127	.00004	*
		2	.05318	.07519	.14484	.31210	.24306	.12536	.03905	.00667	.00054	.00002	*
		3	.05318	.07519	.14484	.59720	.11283	.01535	.00135	.00007	*	*	*
	.60	1	.05018	.05057	.06634	.12133	.20845	.24659	.17557	.06785	.01230	.00001 ,	.00001
		2	.05018	.05057	.06634	.23364	.24243	.20359	.11248	.03516	.00526	.00029	*
		3	.05018	.05057	.06634	.64565	.15241	.03017	.00429	.00037	.00002	*	*

*These values are very small and are equal to zero upto 5 decimal places.

TABLE 3—VALUES OF PARTIAL DIFFERENTIAL COEFFICIENTS OF $E(U_i)$ WITH RESPECT TO α_i AND k FOR DIFFERENT VALUES OF k AND T=10 YEARS*

{LEVEL 1 OF 'α_i' MEANS (.95, .95. . .95)} AND LEVEL 2 MEANS

{(.95, .95 .95 .80 .80. . .80)}

With respect to	Level of α_i	*k*	*Last Parity*					
			1	2	*3*	*4*	5	*6*
i	1	.40	-9.0142	-3.1760	-0.9964	-0.2918	-0.0057	-0.0000
		.60	-14.5833	-7.5173	-2.1617	-0.5802	-0.1303	-0.0900
	2	.40	-9.0142	-3.1760	-0.8550	-0.2886	-0.0056	-0.0003
		.60	-14.5833	-7.5173	-1.5772	-0.5215	-0.1274	-0.0188
k	1	.40	5.5864	2.0149	0.2230	-0.4378	-0.3631	-0.0622
		.60	8.6310	4.4257	1.1624	-0.7240	-0.3214	-0.0554
	2	.40	5.5864	1.8405	1.0435	-0.1879	-0.1183	-0.0322
		.60	8.6310	4.2301	2.3690	-0.2953	-0.1826	-0.0422

*Values corresponding to parity seven have been ignored since they depend on very small proportion.

and the value of *h*, the closed birth intervals sum to the nearest point of the observation period, no matter howsoever compressed.

5. The partial differential coefficients of the mean open birth interval associated with parity 'i' with respect to a_i and *k* as given in Table 3 measure the relative changes in open birth interval with respect to a_1 keeping *k as* constant and with respect to *k* keeping a_i as constant. Only two levels of secondary sterility have been considered for illustra tion. With increasing parity, it is seen that mean open birth interval shows positive changes with respect to a_1. However, the differential coefficient of the mean open birth interval with respect to *k* changes its sign after parity 3, which seems to be the modal parity for women with marital duration of 10 years and with *h* equal to 1.25 years. This means that for given marital duration, the fecundability parameter *k* has positive association with the open birth interval for initial parities but this changes for the latter parities.

5. Conclusions

This study is illustrative in nature. Both the levels of secondary sterility and the fecundability play important role in determining the open birth interval. Though the length of amenorrhoea period can affect very much the open birth interval, it can be assumed constant from parity to parity for a Woman as a first approximation. Or we can deal with such cases by omitting the .women who are either in gestation or in amenorrhoea from the study while collecting data from actual population. Even incidence of foetal losses would have effect of increasing the open birth interval. Their effect can be ascertained separately by introducing two of more types of pregnancies. It is expected that with the increase in the marital duration, the mean open birth intervals for different parities might go on increasing but for the same marital duration they would decrease with the increase in parity since the closed birth intervals would attain stabilising trend and so will their complements. A study of parity progression probabilities would explain the situation which leads to insensitivity of open-birth interval as an index of current fertility for women with higher parities because a very few women of a cohort attain the extreme higher parities. Moreover, majority of the women are tied up with modal parity where the open birth interval seems to be quite sensitive with respect to secondary sterility as well as to fecundability.

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